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T.Y.B.Sc. (Sem.-VI) Examination March - 2026

Mathematics Paper : XIX MTH-603- Old Course Real Analysis-Ill

Time: 2 Hours] **[Total Marks: 50**

સૂચના : / Instructions

(1) ઉપરોક્ત દર્શાવેલ વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fill up strictly the above details on your answer book
(2) All questions are compulsory.
(3) Figures to the right indicate marks of the questions.
(4) Follow usual notations.

Seat No.:

Student's Signature

Q.1. Answer the following questions (Any Five): **10**

1. Prove that a sequence { 1,1,1,1,1,1,, ... } is $(C, 1)$ summable.
2. Define : Regular Summability Method.
3. Let $f_n(x) = x^n$; $0 \leq x \leq 1$. Does a sequence $\{f_n\}_{n=1}^{n=\infty}$ converge uniformly on [0,1]? Justify your answer.
4. Define: Uniformly convergence sequence of functions.
5. Show that every singleton set is of measure zero.
6. Let $\sigma = \left\{0, \frac{1}{2}, 1\right\}$ be a subdivision of [0,1], then find the refinement of σ.
7. If $f \in R\,[a, b]$ and $\lambda \in R$, then prove that $\int_a^b -f = -\int_a^b f$.
8. Evaluate : $\lim\limits_{n \to \infty} \left[\frac{1}{n+2} + \frac{1}{n+4} + \frac{1}{n+6} + \frac{1}{n+8} + \cdots + \frac{1}{n+2n}\right]$.
9. If $F(x) = \int_0^x \sqrt{t + t^6}\, dt\ (x > 0)$, then find $f(4)$.

Q.2. Answer the following (Any two): **10**

a. If the sequence of real numbers $\{S_n\}_{n=1}^{n=\infty}$ converges to L, then prove that it is also $(C, 1)$ summable to L.
b. Prove that the sequence –1, 2, 2, –1, 2, 2, –1, 2, 2 ... is $(C, 1)$ summable. Show that the series $\sum_{n=1}^{n=\infty} (-1)^n$ is $(C, 1)$ summable to $\frac{-1}{2}$.
c. Prove that a sequence 1, –1, 1, –1, 1, –1, 1, –1... $(C, 2)$ summable to 0.

Q.3. Answer the following (Any two): **10**

a. Let $\{f_n\}_{n=1}^{n=\infty}$ be a sequence of continuous real valued functions in $R[a, b]$; which converges uniformly to the function f on $[a, b]$. Then prove that $f \in R[a, b]$.
b. State and prove Cauchy criterion for uniformly convergence sequence of functions.
c. Let $\{f_n\}_{n=1}^{n=\infty}$ be a sequence of continuous real valued functions; that converges uniformly to the function f on $[a, b]$. If for each, $n \in I$, $F_n(x) = \int_a^x f_n(t)dt$; $(a \leq x \leq b)$, then prove that $\{F_n\}_{n=1}^{n=\infty}$ converges uniformly on $[a, b]$.

Q.4. Answer the following (Any two) : **10**

a. Show that the set $\{a, b, c, d\}$ of real numbers is of measure zero.

b. Let $f(x) = x; x \in [0,1]$ and let $\tau = \left\{0, \frac{1}{3}, \frac{2}{3}, 1\right\}$ be any subdivision of $[0,1]$.

Then compute $U[f ; \tau]$ and $L[f ; \tau]$.

c. If f is continuous real valued function on $[a, b]$, then prove that $f \in R[a,b]$.

Q.5. Answer the following (Any two) : **10**

a. Let f be a continuous real valued function on $[a, b]$ and if $F(x) = \int_a^x f(t)dt; x \in [a, b]$, then prove that F is continuous on $[a, b]$.

b. State and prove Mean Value Theorem for integrals.

c. If $f \in R[a,b]$, then prove that $|f| \in R[a,b]$ and $\left|\int_a^b f\right| \leq \int_a^b |f|$.